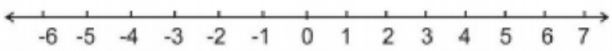
SHRING SHEFFIELD SCHOOL PATNA

Name:____________________________ Class: IX Sub: Math

CHAPTER – 1

NUMBER SYSTEMS

1. Rational Numbers
2. Irrational Numbers
3. Real Numbers and their Decimal Expansions
4. Operations on Real Numbers
5. Laws of Exponents for Real Numbers



- Natural numbers are - 1, 2, 3, ... denoted by N.
- Whole numbers are - 0, 1, 2, 3, ... denoted by W.
- Integers - .... -3, -2, -1, 0, 1, 2, 3, ... denoted by Z.
- Rational numbers - All the numbers which can be written in the form r/s $p/q,$ are called rational numbers where p and q are integers.
- Irrational numbers - A number s is called irrational, if it cannot be written in the form p/q where p and q are integers and
- The decimal expansion of a rational number is either terminating or non-terminating recurring. Thus we say that a number whose decimal expansion is either terminating or non-terminating recurring is a rational number.
- The decimal expansion of a irrational number is non terminating non-recurring.
- All the rational numbers and irrational numbers taken together.
- Make a collection of real number.
- A real no is either rational or irrational.
- If r is rational and s is irrational then r+s, r–s, r.s are always irrational numbers but r/s may be rational or irrational.
- Every irrational number can be represented on a number line using Pythagoras theorem.
- Rationalization means to remove square root from the denominator.

  $\frac{3+\sqrt{5}}{\sqrt{2}}$ to remove we will multiply both numerator & denominator by $\sqrt{2}$ $\frac{1}{a \pm \sqrt{b}}$ its rationalization factor $a \mp \sqrt{b}$

Class IX Chapter 1 – Number Sustems Maths

Exercise 1.1 Question

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers ≠ 0?
and q

Answer:

Yes. Zero is a rational number as it can be represented as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ etc.

Question 2:

Find six rational numbers between 3 and 4.

Answer:

There are infinite rational numbers in between 3 and 4.

3 and 4 can be represented as $\frac{24}{8}$ and $\frac{32}{8}$ respectively.

Therefore, rational numbers between 3 and 4 are $\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$

Question 3:

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$
Answer:

There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

$$\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$$
$$\frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

numbers between $\frac{3}{5}$ and $\frac{4}{5}$

Therefore, rational are $\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

Question 4:

State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Answer:

(i) True; since the collection of whole numbers contains all natural numbers.

(ii) False; as integers may be negative but whole numbers are positive. For example: −3 is an integer but not a whole number.

(iii) False; as rational numbers may be fractional but whole numbers may not be. For example: $\frac{1}{5}$ is a rational number but not a whole number.

Exercise 1.2 Question 1:

State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Answer:

(i) True; since the collection of real numbers is made up of rational and irrational numbers.

(ii) False; as negative numbers cannot be expressed as the square root of any other number.

(iii) False; as real numbers include both rational and irrational numbers. Therefore, every real number cannot be an irrational number.

Question 2:

Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Answer:

If numbers such as $\sqrt{4} = 2,\ \sqrt{9} = 3$ are considered,

Then here, 2 and 3 are rational numbers. Thus, the square roots of all positive integers are not irrational.

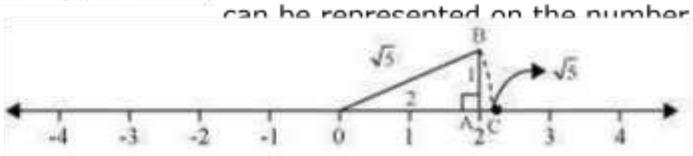
Question 3:

$\sqrt{5}$

Answer:

We know that, $\sqrt{4} = 2$

Show howAnd, $\sqrt{5} = \sqrt{(2)^2 + (1)^2}$ can be represented on the number line.



Mark a point 'A' representing 2 on number line. Now, construct AB of unit length perpendicular to OA. Then, taking O as centre and OB as radius, draw an arc intersecting number line at C.

C is representing $\sqrt{5}$.

has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Answer:

(i) $\frac{36}{100} = 0.36$

Terminating

(ii) $\frac{1}{11} = 0.090909...... = 0.\overline{09}$

Non-terminating repeating

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

Terminating

(iv) $\frac{3}{13} = 0.230769230769.... = 0.\overline{230769}$

Non-terminating repeating

(v) $\frac{2}{11} = 0.18181818........ = 0.\overline{18}$

Non-terminating repeating

(vi) $\frac{329}{400} = 0.8225$

Terminating

You know that $\frac{1}{7} = 0.\overline{142857}$ Question 2:

$\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$

Exercise 1.3 Question 1:

Write the following in decimal form and say what kind of decimal expansion each . Can you predict what the decimal expansion of are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.] Answer:

Yes. It can be done as follows.

$$\frac{2}{7}=2\times\frac{1}{7}=2\times0.\overline{142857}=0.\overline{285714}$$

$$\frac{3}{7}=3\times\frac{1}{7}=3\times0.\overline{142857}=0.\overline{428571}$$

$$\frac{4}{7}=4\times\frac{1}{7}=4\times0.\overline{142857}=0.\overline{571428}$$

$$\frac{5}{7}=5\times\frac{1}{7}=5\times0.\overline{142857}=0.\overline{714285}$$

$$\frac{6}{7}=6\times\frac{1}{7}=6\times0.\overline{142857}=0.\overline{857142}$$

Question 3:

Express the following in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Answer:

(i) $0.\overline{6}=0.666...$

Let x = 0.666...

10x = 6.666...

10x = 6 + x

9x = 6

$x=\frac{2}{3}$

(ii) $0.4\overline{7}=0.4777.....$

$=\frac{4}{10}+\frac{0.777}{10}$

Let x = 0.777...

10x = 7.777...

10x = 7 + x

$x=\frac{7}{9}$

$$\frac{4}{10}+\frac{0.777...}{10}=\frac{4}{10}+\frac{7}{90}$$

$$=\frac{36+7}{90}=\frac{43}{90}$$

(iii) $0.\overline{001}=0.001001...$

Let x = 0.001001...

1000x = 1.001001...

1000x = 1 + x

999x = 1

$x=\frac{1}{999}$

Question 4:

Express 0.99999...in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer:

Let x = 0.9999...

10x = 9.9999...

10x = 9 + x

9x = 9 x =

1

Question 5:

What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Answer:

It can be observed that,

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$.

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer:

Terminating decimal expansion will occur when denominator q of rational number $\frac{p}{q}$ is either of 2, 4, 5, 8, 10, and so on…

$$\frac{9}{4} = 2.25$$

$$\frac{11}{8} = 1.375$$

$$\frac{27}{5} = 5.4$$

It can be observed that terminating decimal may be obtained in the situation where prime factorisation of the denominator of the given fractions has the power of 2 only or 5 only or both.

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.
Answer:

3 numbers whose decimal expansions are non-terminating non-recurring are as follows.

0.505005000500005000005…

0.7207200720007200007200000… 0.080080008000080000080000008…

Question 8:

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.
Answer:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

3 irrational numbers are as follows.

0.73073007300073000073…

0.75075007500075000075… 0.79079007900079000079…

Question 9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 (v) 1.101001000100001…

(i) $\sqrt{23} = 4.79583152331\ldots$

As the decimal expansion of this number is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\sqrt{225} = 15 = \frac{15}{1}$

It is a rational number as it can be represented in $\frac{p}{q}$ form.

(iii) 0.3796

As the decimal expansion of this number is terminating, therefore, it is a rational number.

(iv) 7.478478 … $= 7.\overline{478}$

As the decimal expansion of this number is non-terminating recurring, therefore, it is a rational number.

(v) 1.10100100010000 …

As the decimal expansion of this number is non-terminating non-repeating, therefore, it is an irrational number.

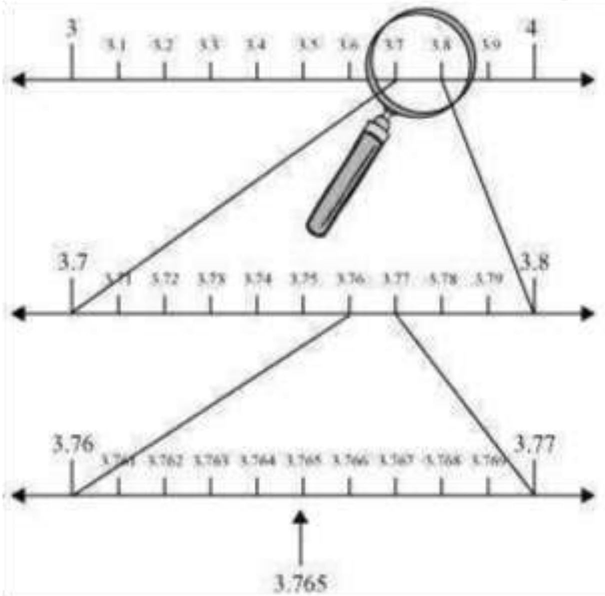
Exercise 1.4 Question

1:

Visualise 3.765 on the number line using successive magnification.

Answer:

3.765 can be visualised as in the following steps.



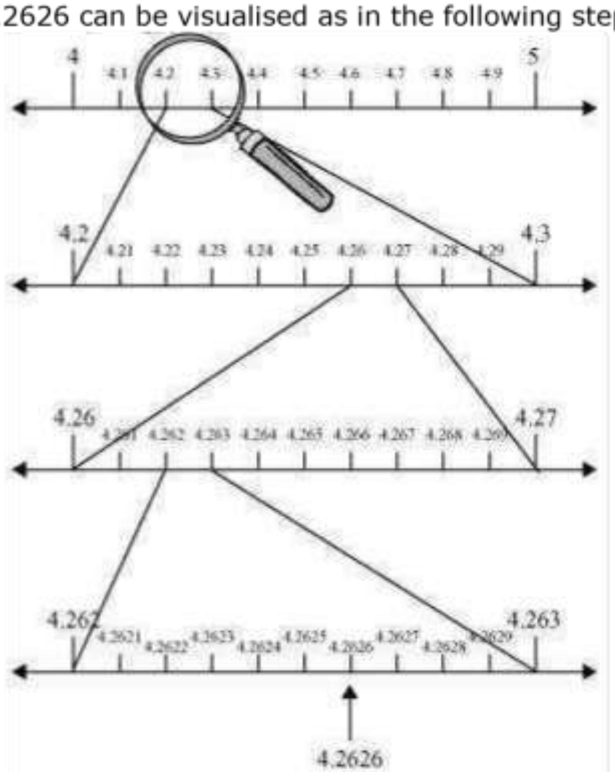
Question 2:

Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Answer:

$4.\overline{26}$ = 4.2626…

4.2626 can be visualised as in the following steps.



Exercise 1.5 Question 1:

1Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Answer:

(i) $2-\sqrt{5}$ = 2 − 2.2360679…

= − 0.2360679…

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

form, therefore, it is a rational

number. form, therefore, it is a

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}=3=\frac{3}{1}$ rational number.

As it can be represented in $\frac{p}{q}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

As it can be represented in $\frac{p}{q}$ As the decimal expansion of this expression is non-terminating non-recurring,

(iv) $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=0.7071067811...$ therefore, it is an irrational number. (v) 2π = 2(3.1415 …)

= 6.2830 …

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

Question 2:

Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ (iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Answer:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=(3)^2-\left(\sqrt{3}\right)^2$

= 9 − 3 = 6

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{2}\right)$

$=5+2+2\sqrt{10}=7+2\sqrt{10}$

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$

= 5 − 2 = 3

Question 3:

Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi=\frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer:

There is no contradiction. When we measure a length with scale or any other instrument, we only obtain an approximate rational value. We never obtain an exact value. For this reason, we may not realise that either c or d is irrational. Therefore, the $\frac{c}{d}$ fraction is irrational. Hence, π is irrational.

Question 4:

Represent $\sqrt{9.3}$ on the number line. Answer:

Mark a line segment OB = 9.3 on number line. Further, take BC of 1 unit. Find the midpoint D of OC and draw a semi-circle on OC while taking D as its centre. Draw a

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

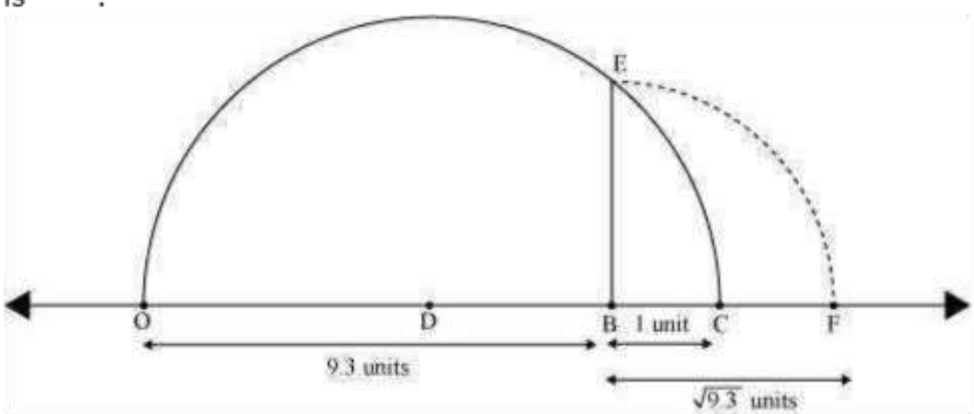
(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Answer:

(i) $\frac{1}{\sqrt{7}}=\frac{1\times\sqrt{7}}{1\times\sqrt{7}}=\frac{\sqrt{7}}{7}$

perpendicular to line OC passing through point B. Let it intersect the semi-circle at E.

Taking B as centre and BE as radius, draw an arc in tersecting number line at F. BF is $\sqrt{9.3}$.



Question 5:

Rationalise the denominators of the following:

(ii)
$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\left(\sqrt{7}-\sqrt{6}\right)}\frac{\left(\sqrt{7}+\sqrt{6}\right)}{\left(\sqrt{7}+\sqrt{6}\right)}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\frac{\sqrt{7}+\sqrt{6}}{1}=\sqrt{7}+\sqrt{6}$$

(iii)
$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{1}{\left(\sqrt{5}+\sqrt{2}\right)}\frac{\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}-\sqrt{2}\right)}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv)
$$\frac{1}{\sqrt{7}-2}=\frac{1}{\left(\sqrt{7}-2\right)}\frac{\left(\sqrt{7}+2\right)}{\left(\sqrt{7}+2\right)}$$

$$=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$$

Exercise 1.6 Question 1:

Find:

(i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Find:

(i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Answer:

Answer:

(i)

$$64^{\frac{1}{2}} = \left(2^6\right)^{\frac{1}{2}}$$

$$= 2^{6\times\frac{1}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^3 = 8$$

(ii)

$$32^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}}$$

$$= (2)^{5\times\frac{1}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^1 = 2$$

(iii)

$$(125)^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}}$$

$$= 5^{3\times\frac{1}{3}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 5^1 = 5$$

Question 2:

(i)

$$9^{\frac{3}{2}} = \left(3^2\right)^{\frac{3}{2}}$$

$$= 3^{2\times\frac{3}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 3^3 = 27$$

(ii)

$$(32)^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}}$$

$$= 2^{5\times\frac{2}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^2 = 4$$

(iii)

$$(16)^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}}$$

$$= 2^{4\times\frac{3}{4}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= 2^3 = 8$$

(iv)

$$(125)^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{1}{\left(5^3\right)^{\frac{1}{3}}}$$

$$= \frac{1}{5^{3\times\frac{1}{3}}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{1}{5}$$

Question 3:

Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Answer:

(i)

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} \qquad \left[a^m.a^n = a^{m+n}\right]$$

$$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

(ii)

$$\left(\frac{1}{3^3}\right)^7 = \frac{1}{3^{3\times7}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{1}{3^{21}}$$

$$= 3^{-21} \qquad \left[\frac{1}{a^m} = a^{-m}\right]$$

(iii)

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right]$$

$$= 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$$

(iv)

$$7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times8)^{\frac{1}{2}} \qquad [a^m.b^m = (ab)^m]$$

$$= (56)^{\frac{1}{2}}$$