Name:____________________________ Class: VIII Sub: Math

Chapter – 01

Rational Numbers

- Rational numbers are **closed** under the operations of addition, subtraction and multiplication.
- The operations addition and multiplication are
  (i) **commutative** for rational numbers.
  (ii) **associative** for rational numbers.
- The rational number 0 is the **additive identity** for rational numbers.
- The rational number 1 is the **multiplicative identity** for rational numbers.
- The additive inverse of the rational number $\frac{a}{b}$ *is* $\frac{a}{b}$ *and vice – versa.*
- The **reciprocal or multiplicative inverse** of the rational number $\frac{a}{b}$ *is* $\frac{c}{d}$ *if* $\frac{a}{b} \times \frac{c}{d} = 1$.
- Distributivity of rational numbers: For all rational numbers a, b and c, a(b + c) = ab + ac and a(b – c) = ab – ac
- Rational numbers can be represented on a number line.
- Between any two given rational numbers there are countless rational numbers. The idea of mean helps us to find rational numbers between two rational numbers.
- **Positive Rationals:** Numerator and Denominator both are either positive or negative. Example: $\frac{4}{7}, \frac{-3}{-4}$
- **Negative Rationals:** Numerator and Denominator both are of opposite signs. Example: $\frac{-2}{11}, \frac{4}{-9}$
- **Additive Inverse:** Additive inverse (negative) $\frac{a}{b} + \frac{-a}{b} = \frac{-a}{b} + \frac{a}{b} = 0$. $\frac{-a}{b}$ is the additive inverse of $\frac{a}{b}$ and $\frac{a}{b}$ is the additive inverse of $\frac{-a}{b}$.
- **Mulitiplicative Inverse (reciprocal):** $\frac{a}{b} \times \frac{c}{d} = 1 = \frac{c}{d} \times \frac{a}{b}$ where $\frac{c}{d}$ is the reciprocal of $\frac{a}{b}$. Zero has no reciprocal. The reciprocal of 1 is1 and of -1 is -1.

Class 7 Maths Integers Introduction

Natural Numbers

- Numbers that come naturally to us are Natural numbers.For example, 1,2,3, 4,…

Count the number of fingers in your hand 1,2,3,4……10, these are natural numbers.

What exist below 1?

Suppose you have 5 chocolates and now you give one chocolate to one of your friend, now you are left with four, similarly you distribute remaining four to other friends, hence you are left with no chocolate or zero chocolate.

Whole Numbers

- Numbers starting from 0,1,2,3,4,5,6……………. are called Whole numbers

Note: The whole numbers start with 0 while natural numbers start with 1,2,3,4..

What exist below 0?

- The numbers below 0 are -1, -2, -3,-4,-5,-6,-7……….. .

Examples: Suppose you borrow one chocolate from your elder brother then you will have one chocolate and that should be counted as -1.
In the Antarctica region the temperature goes well below 0, the temperature usually over there is -10

<u>What are integers?</u>

Collection of all positive(1,2,3,4) and negative numbers(-1,-2,-3..) including 0 are the ***integers***.

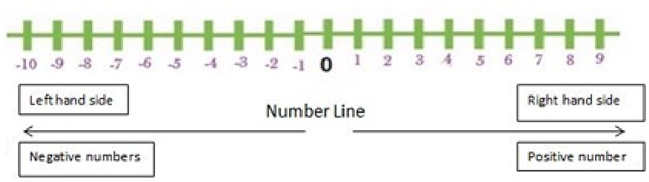
<u>Note</u>: 0 is neither a positive nor a negative integer.

Question: Which number is larger?

10 or 18
-10 or 18
-10 or -18

Solution:
a) 18 is larger than 10.
b) 18 is larger than -10
c) -10 is larger than -18,
These can be proved on a number line

Integer on number Line



Note: The numbers on right hand side are always bigger than the numbers on the left hand side.
For example:

25 > 15 (as 25 lies more on right hand side).
-25 < 15 (as 15 lies more on right hand side).
-15 > -25 (as -15 lies more on right hand side).

- To represent a number on number line we mark the circle on the number line, suppose you need to represent -2, 0 and 2 on number line you put circle over the number.

Rules for Operation on number line

- To Add a +ve number, move right
- To Add a –ve number, move left
- To Subtract a +ve number, move left
- To Subtract a –ve number, move right

Ex: 5+(-3) = 2 (From 5 move 3 jumps on left side, we get 2).
4+(-5)=-1 (From 4 move 5 jumps on left side, we get 2).
0+(-8)=-8 (From 0 move 8 jumps on left side, we get -8)
6+8= 14 (From 6 move 8 jumps on left side, we get 14)

Tip: Subtract the number put the sign of bigger number.

Properties of Integer

Closure Property:

- For any two integers a and b, a+b is always an integer
- For any two integers a and b, a-b is always an integer
- Hence, Addition and Subtraction follows closure property.
  Example: 1+(-15)=14
  1+15=16
  2-(-5)=7
  2-5= -3 .

Commutative Property:For any two integers a and b, a+b=b+a

But, this is NOT true for Subtraction.

i.e. a-b b-a.

Example:

Statement 1	Statement 2	Inference
3 + 4 =7	4 + 3 = 7	Both statements are equal
−15 + (−10) = -25	(−10) + (−15) = -25	Both statements are equal
3 + 12 =15	12 + 3 = 15	Both statements are equal
3-(-5)=9	-5-3=-8	Both statements are different
2-5=-3	5 − 2=3	Both statements are different

- ***Associative Property***:

For three integers a, b and c, [a+b]+c = a+[b+c]
But, this law doesn't hold true for Subtraction.
i.e.   a-(b-c) (a-b)-c
Example:

Statement 1	Statement 2	Result
2+[5+3]=10	[2+5]+3=10	Both statementsare equal
8+[-2+(-3)]=3	[8+(-2)]+(-3)=3	Both statementsare equal
8-[(-2)-(-3)] = 8-(-2+3) = 7	[8-(-2)]-(-3) = 10-(-3) = 13	Both statementsare different

Multiplication of an integer

Multiplication of positive and negative integer

- We should remember when we multiply,
- a xb = ab i.e two positive integer when multiply gives positive integer.
- (-a)x(-b)=ab i.e two negative integer when multiply gives positive number.
- (-a)x(b)=-ab i.e one positive and one negative integer when multiply gives negative number.

Tip: Find the product then a give sign according to the case mentioned above.

Question: Multiply the following numbers

(-2)x(-3)
(-3)x6
2x4
4x(-6)

Solution: a. (-2)x(-3) = 6
b. (-3)x6 = -18
c. 2x4 = 8
d. 4x(-6) = -24

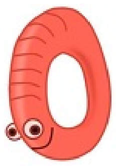
Tip: Multiply the numbers then put the sign accordingly

Multiplication of three or more integers

- Negative-positive-negative gives positive result
  Example: -2x3x(-4)=24
- Negative-Negative-Negative gives Negative result
  Example: -2x(-3)x(-4)= - 24
- Negative-positive-negative gives positive result
  Example: -2x3x(-4)=24

Note: If the numbers of negative sign is even then the sign is positive and if the number of negative sign is odd then the sign of a number is negative.

Multiplication by 0:



Any number when multiplied with 0 always gives 0 value.
Example: 0x6=0
0x5=0

0x(-2)=0

Multiplicative Identity(1):
When we multiply 1 with the number the result will be the number itself i.e. ax1=a.
Example: 1 x 3 = 3
2 x 1 = 2
3 x 1 = 3
-10 x 1 = -10

Note: When we multiply -1 with the number the sign changes.
Example: 2x -1 = -2
-2 x -1 = 2

Multiplication Property:

- ***Closure under Multiplication:*** If we multiply two integers “a, b” the product of two integers will always an integer. The below example proves that.

Statement-1	Inference
2x3=6	Result is Integer
3x(-7)=-21	Result is Integer
-2x(-10)=20	Result is Integer

- ***Commutative of Multiplication:***
  For any two integers a and b, axb= bxa.
  Example:

Statement 1	Statement 2	Inference
3× (− 4) = −12	(− 4) × 3 = −12	Both statementsare equal
(−15) × (−10) = 150	(−10) × (−15) = 150	Both statementsare equal
(−30) × 12 = -360	12 × (−30) = -360	Both statementsare equal

- ***Associative property of Multiplication:***
  If you are multiplying three integers a, b and c, (a x b) x c = a x (b x c)
  Example:

Statement 1	Statement 2	Inference
[(−3) × (−2)] ×5 = 30	(−3) × [(−2) × 5] = 30	Both statements are equal
[(7) × (− 6)] × 4 = -168	7 × [(− 6) × 4] = -168	Both statements are equal

[(7) × (3)] × 2 = 42	7 × [(3) × 2] = 42	Both statements are equal

- ***Distributive Property:*.**
  For three integers a, b and c,
  a x (b + c) = a x b + a x c
  a x (b - c) = a x b − a x c

Example:

Statement 1	Statement 2	Inference
18 × [7 + (−3)] = 72	[18 × 7] + [18 × (−3)] = 72	Both statements are equal
(−21) × [(− 4) + (− 6)] = 210	[(−21) × (− 4)] + [(−21) × (− 6)]=210	Both statements are equal
18 × [7 -6] = 18	[18 × 7] - [18 × (6)] = 18	Both statements are equal

Division of an Integer:

Division refers to splitting into equal parts.
Example Dividing 4 ice-creams into two parts,

We get,

2 ice-creams in each part

i.e 4/2=2

Division of a negative integer by a positive integer

When we divide a positive integer by a negative integer, we first divide them as whole numbers and then put a minus sign (–) before the quotient. That is, we get a negative integer.
In general, for any two positive integers a and b

a (-b)= (-a) b where b 0

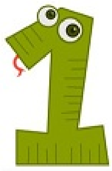
Example: 125 (-25) = -5
-64 8 = 8

Division by zero(0)

- When we divide a number by zero the result is undefined i.e.a 0= undefined

- When 0 is divided by any number the result is zero(0) itself
  Example: 5 0 = undefined
  0 7= 0
  0 (-2)= 0

Division by one(1)



When we divide a number by 1 the result is the number itself

Example: 27 1 =27

-12 1 = -12

20 1 =20

Division Property:

- Commutative Property: In Division"a b b a". Hence, division does not follow Commutative Property
  Example: -15 3 = -5 But, 3 = 1/5 they are not equal
- Associative Property: Division does not follow Associative Property.

Example: 15 [(-3) (-3)] = 15 1 = 5

But, [15 (-3)] (-3) = 5/3Both statements are not equal

- Distributive Property: For three integers "a [b+ c] a b + c".
  Hence division does not follow Distributive Property.

Example: 15 [3+2] = 15 5 = 3

But, 15 +15 2 =5+ 15/2
Both statements are not equal